The classical theorists resemble Euclidean geometers in a non-Euclidean world who, discovering that in experience straight lines apparently parallel often meet, rebuke the lines for not keeping straight – as the only remedy for the unfortunate collisions which are occurring. Yet, in truth, there is no remedy except to throw over the axiom of parallels and to work out a non-Euclidian geometry.


**HOW TO BLOW UP YOUR HEDGE FUND**

The partners of Long-Term Capital Management included Nobel Prize winners and PhDs in finance, economics, maths and physics. Having poached Wall Street’s savviest traders and combined them with what no less a figure than William Sharpe declared to be “probably the best academic finance department in the world” (Siconolfi and Raghavan 1998), LTCM set about deploying the most sophisticated financial models ever devised. Key among these models was value-at-risk (VaR).

The VaR methodology was integral to LTCM’s investment strategy. LTCM viewed portfolio construction as an optimisation problem centred on maximising returns while minimising variance (which in VaR terms is the same thing as minimising “risk”). The end goal of LTCM’s process was to produce superior “risk-adjusted” returns over the long term – high returns with low variance.

From LTCM’s inception in 1994 through 1997, the strategy...
worked, generating a compound annual return over the four-year period of nearly 30% / year, with a variance lower than that of the broad US stock market. The fund appeared to be a wildly successful marriage of theory and practice. The period of 1994 to 1997 only served to confirm the partners’ faith in their models.

Despite the complexity and diversity of LTCM’s portfolio, the partners believed they had distilled their risk profile down to a few simple numbers that could be succinctly captured in a VaR analysis. In a fit of hubris, LTCM disclosed these VaR analyses to their investors, indicating a daily standard deviation of US$45 million and a monthly standard deviation of US$206 million, against total capital of US$5 billion (Jorion 2000). These figures were followed by the declaration of a 99% confidence level that the fund’s losses would not exceed US$105 million on a daily basis, or US$480 million on a monthly basis.

However, in the summer of 1998, LTCM’s “confidence levels” began to break down, at first slowly and then rapidly. In May, the fund lost US$310 million. In June it lost US$450 million, already nearing LTCM’s partners’ 99% confidence level of what losses could not exceed. Still, through June these losses could be understood within the well-defined limits of the model. After four very strong years, it stood to reason that LTCM would experience losses sometime, but that the fund would quickly return to profitability.

Such expectations proved unfounded. Losses for the month of August exceeded US$1,700 million, an 8.3-sigma event according to LTCM’s models. Assuming a normal (Gaussian) distribution, as LTCM did, an 8.3-sigma event should occur approximately once every 80 trillion years. It began to dawn on the partners that something might have gone wrong in their risk modelling.

The losses didn’t come in the form of a gradual bleed. On August 21, 1998, the portfolio lost US$550 million – in a single day. According to LTCM’s VaR model, August 21 represented a 12.2-sigma event. The difference between an 8-sigma event and a 12-sigma event is such that the 8-sigma event, which should occur only once every tens of trillion years, should itself occur billions of times before even a single 12-sigma event occurs. Assuming a normal distribution, a 12.2-sigma event should be rare enough that it essentially breaks the model.
It can be tempting to just ignore this sort of event, except that just one month later LTCM lost another US$550 million in a single day, on September 21. The reality began to sink in that the precise maths that went into and came out of the VaR model – the historical sampling, the covariance calculations, the 99% confidence levels – had proved deeply misleading about the real nature of the risks LTCM was taking. By the end of September, LTCM had lost 92% of its partners’ capital, more than US$4 billion dollars, and almost certainly would have lost it all had the Federal Reserve not intervened to force an orderly liquidation (Figure 2.1).

Subsequent events have proved that 1998 was not a one-off “perfect storm” that we can expect never to see again. The market action that bankrupted LTCM in 1998 – ballooning credit spreads and spiking equity volatility – was only a fraction of the magnitude of what happened in 2007–2008. As Eric Rosenfeld, one of the LTCM partners, conceded in a 2009 presentation at the Sloan School of Management, if LTCM had somehow survived 1998, its collapse in 2007–2008 would have been orders of magnitude more spectacular.

Not only did LTCM’s VaR models fail to prepare them for a market event that in retrospect appears to occur about once per decade,
but LTCM’s VaR models encouraged a portfolio construction that rendered the firm uniquely vulnerable to that event. The case of LTCM is instructive because it reveals the three fundamental weaknesses inherent to the VaR methodology: (1) the false assumption of a normal distribution and therefore a unique vulnerability to the problems presented by fat tails; (2) a naïve equation of variance with “risk”; and (3) the problems inherent in using the past to predict the future.

The remainder of this chapter will discuss each of these three points.

NOT ALL DISTRIBUTIONS ARE NORMAL

Some data; non-linearity of returns

The first question that needs to be addressed is why, in practice, firms relying on VaR fail so frequently.

Some historical data helps to clarify the picture. To take the most familiar example, since its inception in 1896, the Dow Jones Industrial Average (Figure 2.2) has had more than 29,000 trading days. Over this period, the index has demonstrated a daily standard deviation of 1.16%.

![Figure 2.2 DJIA: 1896–2011](source: Data taken from Bloomberg; authors own composition)
Assuming a normal (Gaussian) distribution of returns, as the VaR model does, we should expect to find 98% of the daily moves of the index to fall within a range of 2.33-sigma (in this case a daily change of about +/- 2.69% around the mean daily return of 2.6bp). In other words, the theory predicts that the index should have exhibited either a positive or negative move around the mean of a magnitude greater than 2.69% about 574 times – in theory. In the historical sample, the actual number was 918 (Table 2.1).

Again, assuming a normal distribution, over the 116-year history of the index we should expect to find about 80 observations beyond the 3-sigma level. In the 116-year sample, we observe 480 such instances.

Assuming a normal distribution, over the same 116-year period we should expect to find about two observations beyond the 4-sigma level. In the sample, we observe 202.

Assuming a normal distribution, we should expect to find a 5-sigma event approximately once every 7,000 years, a fair amount longer than the 116-year history of the index. In the historical sample, we observe 87 such instances over the 116 years, a little under one per year.

Assuming a normal distribution, we should expect to find a 6-sigma event once every 2 million years. In the sample, we observe 48, a little under one every other year.

Assuming a normal distribution, we should expect to find a 7-sigma event once every 1.5 billion years. In the sample, we observe 27, about once every four years.

Eight-sigma events should not occur even once over countless iterations of the history of the universe, yet in the sample we observe 20, about once every six years.

By the time we reach beyond 10-sigma events, the normal distribution predicts such events to be so infinitesimally rare that the probabilities are no longer meaningful to us. And yet in practice we’ve experienced 10-sigma events nine times in the last 116 years, or a little less than once per decade.

At the very far ends of the tails, we have December 14, 1914, an 18-sigma event, and of course October 19, 1987, a 20-sigma event. For practical purposes such events fall outside the bounds of the model.
Table 2.1 1-Day Events in the DJIA: 1896–2011

<table>
<thead>
<tr>
<th>Sigma</th>
<th>Expected</th>
<th>Observed</th>
<th>Error factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.33-sigma</td>
<td>575</td>
<td>918</td>
<td>1.6</td>
</tr>
<tr>
<td>3-sigma</td>
<td>78</td>
<td>480</td>
<td>6.2</td>
</tr>
<tr>
<td>4-sigma</td>
<td>1.83</td>
<td>202</td>
<td>110.4</td>
</tr>
<tr>
<td>5-sigma</td>
<td>0.0166</td>
<td>87</td>
<td>5,240.9</td>
</tr>
<tr>
<td>6-sigma</td>
<td>0.0000573</td>
<td>48</td>
<td>837,696.3</td>
</tr>
<tr>
<td>7-sigma</td>
<td>7.43243E-08</td>
<td>27</td>
<td>363,272,718.9</td>
</tr>
<tr>
<td>8-sigma</td>
<td>0</td>
<td>20</td>
<td>infinite</td>
</tr>
<tr>
<td>10-sigma</td>
<td>0</td>
<td>9</td>
<td>infinite</td>
</tr>
</tbody>
</table>

Source: Data taken from Bloomberg; authors own composition

Thus, what the theory predicts and what actually happened represent wildly divergent sets of outcomes (Table 2.1). A normal distribution has not been a safe assumption over the last 116 years of the Dow (Figures 2.3 and 2.4).
This by itself isn’t necessarily damning. We need not observe a perfect normal distribution in the real world in order to make use of mathematical tools that assume a normal distribution. After all, the important differences we see between theory and reality occur only at the extremes of the distribution. These extremes – the tails – by definition represent rare events. As outlined above, as the events become more extreme, the error factors increase by orders of magnitude into the trillions and beyond.

But, since these events are rare, the large error factors are equally rare. If instead we shift our focus towards the middle of the distribution, where the bulk of the observations lie, we see that about 80% of the Dow’s trading days fall within 1-sigma, against the theory’s prediction of 67%. Now we are talking about an error factor closer to 20%. Based on this, it might appear that VaR can still be useful for understanding the risks involved in more “normal” markets, even if it is less useful for predicting the frequency of extreme events.

This appearance is misleading. When we are talking about financial market risks, we cannot simply ignore the extreme events in favour of focusing on “normal” markets, because it is the extreme events that drive returns. The power of the tails is enormous, and their impact is not linear. This nonlinearity renders the results of tail analyses deeply counterintuitive.

**Figure 2.4** DJIA: 1896–2011, left tail close-up

Source: Data taken from Bloomberg; authors own composition
Consider for a moment “upside” risk. If we were to remove the top 100 trading days – fat tails every one of them – from the 29,000-day sample, we would also be removing 99.79% of the cumulative return of the index over the last 116 years. In other words, 0.34% of the trading days – those concentrated in the tail – are responsible for more than 99% of the index’s cumulative return (Table 2.2).

To make a completely fair comparison, however, we need to replace the top 100 observed trading days with the top 100 trading days we would expect from a normal distribution of a 29,000-day sample. When we make this swap, the cumulative return over the 116 years drops by 93.5% compared with the actual historical return. The gulf between the normal distribution and the observed reality is so large that the difference between the two over just 100 days changes the cumulative impact of a 29,000-day sample by a factor of 15! (Figure 2.5)

Again, this impact is not linear. If we were to simply remove the top 10 trading days rather than the top 100, we would be removing 67% of the cumulative return over the 116 years. In other words, 0.034% of the trading days represent two-thirds of the cumulative 116 year return (Table 2.2). If we replace these 10 days with the returns predicted by the normal distribution, the cumulative return for the index drops by half.
It is extremely easy to underestimate the power of the tails.

Table 2.2  DJIA: 1896–2011

<table>
<thead>
<tr>
<th>Cumulative impact in the distribution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Top 10 days</td>
<td>66.87%</td>
</tr>
<tr>
<td>Days 11–100</td>
<td>32.92%</td>
</tr>
<tr>
<td>Remaining 29,000 days</td>
<td>0.21%</td>
</tr>
</tbody>
</table>

Thus, even if we can safely assume that VaR works well during the 99% of the time that represents “normal” markets, this 99% of the time when VaR works accounts for less than 1% of the cumulative impact of the distribution. Because VaR is unable to account for what happens in the tails, it is unable to account for what actually drives risk and returns in financial markets. It is not a useful measure.

In terms of “downside” risk, the maths is equally compelling, but far more simple. A risk manager assuming a normal distribution into a VaR model could reasonably design a portfolio to withstand a 5-sigma event based on the expectation that the portfolio would make it through the next 7,000 years without a problem. Based on the actual record, we can expect such a portfolio to blow up approximately every 2.5 years.

This is a deeply flawed model.